HYPOSONIC AERODYNAMICS (FLOWS AT LOW MACH NUMBERS)

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1. Introduction. This work is the result of efforts to understand weakly compressible flow and various types of associated effects of nonstationarity, viscosity, and acoustics. Only perfect gas flows with constant specific heat in the absence of gravity are considered. Right from the very beginning the effects of rotation, Coriolis forces, and electrical and magnetic fields are not considered since the governing equations are the classical Navier–Stokes equations. This work, in the form presented here, may surprise the reader by the absence of a detailed study of any specific problem and instead contains a series of questions which at present require the correct consideration of weak compressibility in the mathematical modeling of different physical phenomena. Discussions are illustrated by some simple problems and for some of them various stages of the solutions are described. The terminology "hyposonic" is suggested for these weakly compressible fluid flows. Thus the present work is presented to a large extent as a program.

Consider the motion of a perfect gas with constant specific heats c_p and c_V, which may be viscous and conducting; these motions are simply referred to as flows. The kinematic description of the given flow is expressed in Eulerian terms: time t and coordinates x_i (i = 1, 2, 3) of the fluid particles in an orthonormalized Cartesian coordinate system. The notation is classical: **u** is the velocity vector with components u_i; p, ρ, and T are the pressure, density, and temperature. It is assumed that the reference parameters for nondimensionalizing are U_∞ for velocity, t_0 for time, L_0 for the position vector, p_∞, ρ_∞, and T_∞ for the thermodynamic quantities. In this case, the equations describing the given flow take the classical from [1] in nondimensional variables and using the same notation for different parameters:

$$\rho\frac{D\mathbf{u}}{Dt}+\frac{1}{\gamma M_\infty^2}\nabla p=\frac{1}{\mathrm{Re}}\left\{\Delta\mathbf{u}+\frac{1}{3}\nabla(\nabla\cdot\mathbf{u})\right\}, \tag{1.1}$$

$$\frac{D\log\rho}{Dt}+\nabla\cdot\mathbf{u}=0,$$

$$\rho\frac{DT}{Dt}-\frac{\gamma-1}{\gamma}\frac{Dp}{Dt}=\frac{1}{\mathrm{Pr}}\frac{1}{\mathrm{Re}}\Delta T+(\gamma-1)\frac{M^2}{\mathrm{Re}}\Phi,\quad p=\rho T,$$

where $D/Dt = S\partial/\partial t + \mathbf{u}\cdot\nabla$, $\Delta = \nabla^2$. In Eqs. (1.1), we assume that the coefficient of bulk viscosity equals zero (Stokes hypothesis) and that the dynamic viscosity coefficient μ_0 and conductivity k_0 are constants. Φ refers to the viscous dissipation (well-known function of **u**). Finally, $\gamma = c_p/c_V$ and $R = c_p - c_V$; $S = L_0/t_0U_\infty$ is the Strouhal number; $M_\infty = U_\infty/u_\infty$ is the Mach number, where $c_\infty^2 = \gamma RT_\infty$; $\mathrm{Re} = U_\infty L_0/(\mu_0/\rho_0)$ is the Reynolds number; $\mathrm{Pr} = \mu_0 c_p/k_0$ is the Prandtl number.

In classical aerodynamics, it is usually assumed that Mach number M_∞ is an important flow parameter at high speeds when it characterizes the compressibility effects [2]. On the other hand, flows at low Mach numbers ($M_\infty \ll 1$) are conventionally associated with incompressibility [3]. However, many studies on the generation of aerodynamic noise, whose initial developments are given in [4, 5] are based on a subtle mechanism over a certain range of flow according to which compressibility plays a significant role in low Mach number flow subject to the condition that observation is made over a considerable distance or for a short interval of time after the initiation of the flow from rest.

Many physical problems arise from the theory of flows at $M_\infty \ll 1$, and the reference [6] may be referred in this connection. We shall discuss here only two examples [we do not consider atmospheric flows which are entirely natural flows at $M_\infty \ll 1$ in which compressibility plays an important role with interactions as a result of Coriolis forces, baroclinicity, and stratification. The reader may refer to [7, 8] and also to [9]. As regards the effects of

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weak compressibility on magnetohydrodynamic flows, certain results could be found in [10]. Finally, it remains to develop an asymptotic theory of the ocean dynamics using a unique approach based on the hypothesis (very realistic) that $M_\infty \ll 1$ for liquids).]: firstly, the flow corresponding to the compression cycle in the internal combustion engine, and secondly, the strongly nonstationary phenomenon of the entry of a high-speed train in a tunnel.

Everything that has been mentioned above stimulates us, at the present time, to consider various implications of the concept of low Mach number flow on the mathematical modeling of many physical phenomena in the Nature surrounding us.

We shall henceforth use the terminology "hyposonic" to characterize these problems on flows at $M_\infty \ll 1$.

2. General Considerations for the Case $S \neq 0$, $Re \neq \infty$. When a flow described by Eqs. (1.1) is considered, it is necessary to specify, in conformity with the physical problem, the initial conditions, conditions at the wall Σ which confines the flow, or possibly, when this region is extended to infinity (external flow), the behavior of the flow at infinity.

It appears that the transition to the limit $M_\infty \to 0$ which generates acoustic waves results in an incorrectly posed (in the classical Adamar sense) Cauchy problem with initial condition specified on the basis of Eqs. (1.1). At present, it is not possible to develop a uniformly valid asymptotic solution (in time and in the entire physical domain occupied by the flow) for weakly compressible flow in the neighborhood of three-dimensional obstacles, starting from the Navier–Stokes equations (1.1) with initial conditions, boundary conditions on the wall Σ, and the behavior of the flow at infinity (external flow). When $S = O(1)$ and $Re = O(1)$, this problem is very complex since it is very difficult to analyze the behavior of the initial and final solutions which describe hyposonic flow at $t = 0$ and at infinity, respectively. On the other hand, it is also necessary to consider the temperature condition on Σ; this condition could be given in the form

$$T = 1 + \tau_0 \Theta \quad \text{on} \quad \Sigma, \tag{2.1}$$

where $\tau_0 = \Delta T_\infty / T_\infty$ and ΔT_∞ is the characteristic change in temperature associated with the temperature field Θ, which is assumed known on Σ.

Taking this fact into consideration, it appears quite natural to consider the dual transition to the limit:

$$M_\infty \to 0 \text{ and } \tau_0 \to 0.$$

There is an approximate expression for the solution of the external flow problem, as M_∞ and τ_0 tend to zero, in the form of a series containing only one small parameter, that satisfies two particular cases [11]: M_∞ is fixed, $\tau_0 \to 0$, $M_\infty \to 0$; τ_0 is fixed, $M_\infty \to 0$, $\tau_0 \to 0$. This description, called intermediate, has a physical meaning that two small parameters M_∞ and τ_0 tend to zero simultaneously in such a manner that the correction for weak compressibility has the same order as the correction that takes the temperature effect into account. This approach is the generalization of the principle of least confluence used in problems with many small parameters, which requires that all parameters tend to zero such that the maximum number of terms are retained in the first approximation.

The case $\tau_0 = \Lambda_0 M_\infty^\lambda$, $M_\infty \to 0$ with fixed Λ_0 of the order of unity is considered in [3] for $S \equiv 0$. It appears that it is then necessary to consider three different situations corresponding to $0 < \lambda < 2$, $\lambda = 2$, and $\lambda > 2$. Naturally, in this case the asymptotic expressions obtained are uniformly valid in the given physical domain.

3. Hyposonic Flow inside a Closed Region. The hyposonic internal flow problem in which the boundary is deformable and subject to a temperature field is considered in [12, 13]. In order to explain the singular nature of the limiting transition $M_\infty \to 0$, we study the behavior of the hyposonic flow with low viscosity in the initial phase of the motion when the nondimensional time is of the order $O(M_\infty)$. This means that a time scale is chosen such that it corresponds to one exit and return of the acoustic wave in the enclosed space. For this, scale differentials in time appear again and, consequently, there is a possibility of satisfying the imposed initial condition; equations of motion are the classical acoustic equations. The eigenmodes are excited as if the wall in the limited region is brought into motion. When this motion of the bounded wall is achieved within the time scale on the order of $O(M_\infty)$, whether it is instantaneous or gradual in this scale, a problem arises as to what happens to these eigenwaves during the period $O(1)$ when it is necessary to study the flow of

the incompressible fluid. However, since acoustic waves pass through the entire initial period without weakening, it is necessary to expect their existence for a prolonged duration. If the formal expansion in terms of M_∞ is carried out in view of this fact, then in order to justify the concept of incompressible flow, it would be necessary to come to the phenomenon consisting of a prolonged existence of acoustic oscillations. In fact, this phenomenon is discussed in detail initially within the framework of a perfect fluid [12] using the multiple-scales technique which requires an infinite number of such scales of rapid times representing different acoustic eigenmodes of the fluid present in the enclosed space at time t. In the first approximation, at $M_\infty \ll 1$, hyposonic flow is a superposition of the mean flow, which is almost incompressible, and acoustic oscillations whose amplitude growth was obtained using the principle of exclusion of terms of the order $O(M_\infty^2)$. It appeared that the amplitude of each mode of oscillation satisfies the ordinary differential equation depending only on the nature of the mean flow. The result that appears to be important to us for possible applications is that the effect of acoustic disturbances reduces at the level of the mean of flow of an almost incompressible fluid to the introduction of an additional term associated with the square of the amplitude of acoustic disturbances in the Bernoulli integral. In the present case, the motion starts from rest and the speed approaches its value $O(M_\infty)$ during the initial phase of the interval $O(M_\infty)$ which leads to an acceleration of the order $O(1)$ during this period whereas they are of the order $O(M_\infty)$ during the following interval of time. The excitation of acoustic waves is associated with this increase in speed during a very short interval of time. If the increase in speed takes place during an interval of time of the order $O(1)$, then acoustic waves appear only in the second approximation and the first approximation results in the classical incompressible flow.

On the other hand, since acoustic waves always damp out as a result of viscous dissipation at the wall, the effect of this friction in the Stokes layer of thickness $O(\sqrt{M_\infty/\mathrm{Re}})$ was computed in [13], for each eigenmode and for each of them the damping rate was obtained. It was observed that the time scale during which the damping of acoustic waves takes place as a result of viscous friction at the wall, assumed cold, was $O(\sqrt{\mathrm{Re}M_\infty})$, which is considerably more than one. Thus, the theory developed in [12, 13] corresponds to the following limiting transition process: M_∞ is fixed, $\mathrm{Re} \to \infty$, $M_\infty \to 0$ since $M_\infty/(1/\mathrm{Re}) \gg 1$. It remains to throw light on one important question concerning the behavior of the Rayleigh layer (see [14]) when $t \geqslant O(1)$. Actually, in the analysis of the initial layer $t = O(M_\infty)$ in a low viscosity fluid there is a Rayleigh layer of thickness $O[(t/M_\infty)^{1/2}]$ at the wall, and, when $t = O(M_\infty)$ and especially since $t = O(\sqrt{\mathrm{Re}M_\infty}) \gg 1$, this layer, if it exists as such, would have occupied the entire enclosed space. Thus, it is important to understand how this Rayleigh layer transforms as $t \geqslant O(1)$. This is a very difficult problem, which in certain general sense is associated with the derivation of the equation of nonstationary compressible boundary layer.

Finally, we note that if the specified temperature at the wall is $O(1)$, then a disagreement is observed between the wall temperature and the fluid temperature in the enclosed region. Consequently, there is necessarily a temperature boundary layer of $O(1)$, whereas the investigation carried out in [13] is based on temperature (and velocity) boundary layers of the order $O(M_\infty)$. This difficulty was circumvented with the assumption made in [13] that the wall is cold.

It remains to consider the general case

$$\mathrm{Re}M_\infty = A_0 = O(1),\ M_\infty \to 0$$

and realize that the Zeitounian and Guiraud theory corresponds to the limiting case $A_0 \to \infty$.

4. Effect of Weak Compressibility on Viscosity. The consideration of the behavior of system (1.1) with condition (2.1), when $\mathrm{Re} \to \infty$, $M_\infty \to 0$, and $\tau_0 \to 0$ simultaneously raises many as yet unsolved problems.

Consider the simple case of the plane stationary flow ($S = 0$, $\partial/\partial x_3 \equiv 0$) of a perfect fluid with constant c_p and c_v about a semiinfinite flat plate, whose surface has a certain temperature ($\Theta \equiv 1$) and which occupies the entire half plane $Ox_1 > 0$ and located in the flow having a uniform velocity U_∞ parallel to the Ox_1 axis. For the particular condition

$$(1/\sqrt{\mathrm{Re}})/M_\infty^2 = 1,\quad \tau_0/M_\infty^2 = \Lambda_0 = O(1)$$

the skin-friction coefficient at the wall of the semiinfinite flat plate is given by the equation (for $\mathrm{Pr} \equiv 1$)

$$c_f \cong \frac{0.664}{\sqrt{\mathrm{Re}_{x_1}}} + \sqrt{x_1}\,\frac{0.664\Lambda_0 + 2f''(0)}{\mathrm{Re}_{x_1}} + \ldots, \qquad (4.1)$$

where $\mathrm{Re}_{x_1} = U_\infty x_1/(\mu_0/\rho_0)$ is the local Reynolds number. The function $f(\eta)$ where $\eta = x_2/\sqrt{x_1}$ is the solution of the problem

$$2f'' + Ff'' + F''f = 4\Lambda_0 F''^2 + (\gamma - 1)[6F'F''^2 - 2F''^2],$$
$$f(0) = f'(0) = f'(\infty) = 0,$$

where $F(\eta)$ is the solution of the classical Blasius equation. We note that the term proportional to $\sqrt{x_1}/\mathrm{Re}_{x_1}$ arises due to the effect of weak compressibility and low viscosity.

It remains to derive a conclusion from (4.1) and in particular generalize the first results given above for an arbitrary obstacle. Besides, it is possible to restrict to the parabolic case, e.g., for which the results of [15-17] are used in the case of an incompressible fluid. This makes it possible to discuss that part which appears in the second approximation of the incompressible boundary layer.

It is also possible to pose the problems on the behavior of the solution of (1.1) as

$$\mathrm{Re} \to 0 \text{ and } \mathrm{M}_\infty \to 0 \qquad (4.2)$$

so that the Knudsen number $\mathrm{M}_\infty/\mathrm{Re} \ll 1$ (it is assumed that the fluid is a continuous medium).

Certain considerations show that the transition to the limit (4.2) must be made with the condition

$$\mathrm{Re}^{1+a}/\mathrm{M}_\infty = O(1), \quad a > 0. \qquad (4.3)$$

If, in particular, it is assumed that $\mathrm{Re}^{\alpha} \equiv \mathrm{M}_\infty/\mathrm{Re} \ll 1$, then as $\mathrm{Re} \to 0$ it is possible to study the solution of Navier–Stokes equation (1.1) in the following form:

$$\mathbf{u} = \mathbf{u}_0 + o(1),\ p = 1 + \mathrm{Re}^{1+2a}(p_1 + o(1)),$$
$$\rho = \rho_0 + o(1),\ T = T_0 + o(1). \qquad (4.4)$$

Then for $\mathbf{u}_0$, p_1, ρ_0, and T_0 we get a limiting system which is written in three parts:

$$\Delta T_0 = 0; \qquad (4.5)$$

$$\rho_0 = 1/T_0; \qquad (4.6)$$

$$\Delta \mathbf{u}_0 = \nabla\pi,\quad \pi = \frac{1}{\gamma}p_1 + \frac{1}{3}\frac{D\log\rho_0}{Dt},$$
$$\nabla\cdot\mathbf{u}_0 + \mathbf{u}_0\cdot\nabla\log\rho_0 = S\partial\log\rho_0/\partial t, \qquad (4.7)$$

where $D/Dt = S\partial/\partial t + \mathbf{u}_0\cdot\nabla$.

Equation (4.5) determines T_0, as soon as the temperature conditions are specified; e.g., at walls, where the temperature is specified, or on cold walls. The relation (4.6) determines the density field ρ_0. Finally, system (4.7) should allow the determination of the velocity field U_0 and the pressure disturbance p_1. We note that the structure of Eqs. (4.7) is close to the structure of classical Stokes equations, though they are far more complex. In view of this fact, it is necessary to consider along with (4.4) the local representation in the neighborhood of the starting time and another representation in the neighborhood of infinity (in the case of external flow problem). We cannot continue the discussion of this model further starting from (4.2)-(4.4) and serious analysis is yet to be done here.

5. Small Disturbance Theory for Hyposonic Flows. Let us discuss the problem of hyposonic motion of a slender body. If, in the general case, the flow upstream is assumed unsteady, then along with the value of M_∞, being the constant characteristic value of the variable Mach number upstream of the flow from the obstacle, there appear two small parameters which are respectively the relative thickness h of the body and a part μ_∞ of the flow nonuniformity at the body associated with the change in Mach number in the upstream region.

Thus, for a perfect fluid, the small disturbance theory for hyposonic flows leads to the problem of explaining the asymptotic behavior of stationary flow of inviscid, nonconducting fluid as a result of a triple transition to the limit:

$$h \to 0,\ \mathrm{M}_\infty \to 0,\ \mu_\infty \to 0 \qquad (5.1)$$

with the condition that these three limits are not independently achieved but are a result of the superposition of two similarity relations:

$$M = H_0 h^{1/2},\ \mu_\infty = M_0 h,\ h \to 0, \tag{5.2}$$

where H_0 and M_0 remain quantities of the order of unity as $h \to 0$. Naturally, in the first approximation (with zeroth order) the classical incompressible flow is violated but starting from the second approximation there appear additional source terms associated with the parameters for hyposonic flow similarity H_0 and M_0. The exact nature of the effect of these terms on the aerodynamic characteristics of the disturbed flow is not known at present in the general case of slender three-dimensional body. We observe that in [18] a detailed analysis has been made for the potential flow past slender three-dimensional wing for the case $M_0 \equiv 0$, to be more precise, this analysis corresponds to the following limiting transition:

$$M_\infty^2 = \widehat{M}_\infty h,\ h \to 0,$$

where $\hat{M}_\infty$ is fixed and is of the order of unity, since the correction for wing thickness is of the same order as the correction for weak compressibility.

Consider again the plane ($\partial/\partial x_3 \equiv 0$) stationary flow ($S = 0$) of inviscid and nonconducting fluid ($Re \equiv \infty$); the thin plane profile is symmetric and continuous along the x_1 axis; the angle of attack is zero so that the flow inclination everywhere is very small. In the plane Ox_1x_2 with the origin at O, the equation for this profile has the form

$$x_2 = h\eta(x_1),\ \eta(0) = \eta(1) = 0. \tag{5.3}$$

For the above assumption for the profile, the wake behind the body, starting from the trailing edge $x_1 = 1$, and the streamline coming to the leading edge $x_1 = 0$ coincide respectively with the straight lines $x > 1$ and $x < 0$. The two unknowns in this problem are the variation $\delta(x_1, x_2)$ of the streamline in the flow disturbed by the slender profile ($h \ll 1$) (5.3) with respect to its position at upstream infinity as $x_1 = -\infty$, and the density disturbance ω appearing in this disturbed flow. These two unknown functions satisfy the Bernoulli integral and the expression that determines the vortex component perpendicular to the plane Ox_1x_2. If the solution is sought in the form

$$\delta = \delta_0 + h^\alpha \delta_\alpha + \ldots,\ \omega = h^\beta \omega_\beta + \ldots,$$

Then it follows from the Bernoulli integral that $\beta = 2$ and then from the expression for the vortex components normal to the plane of the flow, we find that $\alpha = 1$. Following this path we find the second similarity relation (5.2). Thus,

$$\omega_2 = H_0^2 M_\infty^2 (x_2) \frac{\partial \delta_0}{\partial x_2},$$

where $M_\infty(x_2)$ is the variable Mach number upstream of the flow, being a function of x_2. Functions δ_0 and δ_1 are the solutions of the following problems:

$$\frac{\partial^2 \delta_0}{\partial x_1^2} + \frac{\partial^2 \delta_0}{\partial x_2^2} = 0,\ \delta_0(x_1, 0) = \eta(x_1),\ x_1 \in [0,1], \tag{5.4}$$

$$\lim \delta_0 = 0,$$
$$x_1^2 + x_2^2 \to \infty;$$

$$\frac{\partial^2 \delta_1}{\partial x_1^2} + \frac{\partial^2 \delta_1}{\partial x_2^2} = -H_0^2 M_\infty^2 (x_2) \frac{\partial^2 \delta_0}{\partial x_2^2} - M_0 \frac{d}{dx_2}\left(\log M_\infty^2 (x_2)\right) \frac{\partial \delta_0}{\partial x_2}, \tag{5.5}$$

$$\delta_1(x_1, 0) = -\eta(x_1) \frac{\partial \delta_0}{\partial x_2}(x_1, 0),\ x_1 \in [0,1],$$

$$\lim \delta_1 = 0,$$
$$x_1^2 + x_2^2 \to \infty.$$

For the above theory to be correct, the leading edge of the profile should have the shape of an ideal sharp wedge; if this condition is not fulfilled, it is necessary to make a local investigation of the neighborhood of the leading edge, assumed rounded, and matching the local solutions thus obtained with the (basic) solutions of the problem (5.4), (5.5).

The situation becomes somewhat complex when it is desirable to consider small viscosity ($Re \gg 1$). In this case, in addition to the small parameters h, M_∞, and μ_∞, two new small parameters appear. There are the inverse Reynolds number, and the fraction of thermal nonhomogeneity τ_0 of the slender body surface, which is present in the boundary condition (2.1).

Then we should consider instead of (5.1) the multiple transition to the limit:

$$h \to 0,\ M_\infty \to 0,\ \mu_\infty \to 0,\ 1/\sqrt{\mathrm{Re}} \to 0,\ \tau_0 \to 0,$$

where in addition to (5.2) two other similarity relations should also be satisfied

$$\tau_0 = T_0 h,\ 1/\sqrt{\mathrm{Re}} = R_0 h,\ h \to 0,$$

where T_0 and R_0 remain of the order of unity as $h \to 0$.

In this case, for the flow around the slender body as $h \to 0$ we get a flow pattern which is not valid in the neighborhood of the wing surface, where, in accordance with the general rule, it is necessary to investigate the local representation. In a perfect fluid ($T_0 \equiv 0$ and $R_0 \equiv 0$) this difficulty is overcome by studying the behavior of the major part of the solution in the neighborhood of the wing surface. However, in a low viscosity fluid and for the leading edge, this is not possible and, on the contrary, a knowledge of the behavior at infinity of the local representation should make it possible to complete the computation of the boundary layer starting from the leading edge solution obtained from the matching condition.

The slender wing case is considered in [19] but the thickness is much greater than the boundary layer thickness in the incompressible fluid:

$$h \gg 1/\sqrt{\mathrm{Re}} \Leftrightarrow h \ \text{ fixed},\ 1/\sqrt{\mathrm{Re}} \to 0, \ \text{ then } h \to 0.$$

We note that the flow analysis in the neighborhood of the order of $O(\mathrm{Re}^{-3/8})$ at the trailing edge indicates a mechanism of singular matching in the triple deck according to [20, 21] between the boundary layer and the potential flow which is mainly manifested in the form of the effect of a two-sided copper. This analysis, in which the ratio $\mathrm{Re}^{-1/4}/h$ enters, is carried out in [22] for an incompressible fluid, with a description of the separated flow including two recirculation zones with constant vorticity; however, the effect of weak compressibility on this system is not known.

In general, it is necessary to be well aware of the fact that we, at present, know very little about problems concerning the effect of weak compressibility on the flow of weakly viscous fluid near a slender wing, in spite of the obvious interest in such studies due to their application in various real physical phenomena.

6. The Case $\mathrm{Re} \equiv \infty$. External Nonstationary Problem. In this case, the starting point is the Euler equations for nonstationary compressible fluid flows. Perfect fluid with constant specific heats c_p and c_V extend up to infinity in all directions and is bounded inside by a finite closed surface Σ. At large distances from Σ the fluid is uniformly at rest. It is appropriate to use a system of coordinates fixed to the fluid at rest with respect to which Σ is subject to an arbitrary displacement but at sufficiently small speed; in a more general case, the fluid velocity at each point is assumed "very small" when compared to the local speed of sound (hyposonic motion). The solution of Euler equations (at the level of equations (1.1), a transformation is made to the limit $\mathrm{Re} \to \infty$ as t and x_i are fixed) should satisfy the initial conditions (fluid at rest at $t = 0$) and boundary conditions along with the initial conditions on the one hand at infinity, where the fluid is uniformly at rest, and on the other hand, at the surface Σ. It is assumed that these conditions are sufficient for the uniqueness of the solution, if, obviously, the location and the shape of Σ are prescribed. Only in [23, 24] it was mentioned that the nonstationary mechanism of sound generation by hyposonic flows of a perfect fluid, being the result of finite displacement of obstacles in an infinite atmosphere, was obtained with the application of matched asymptotic expansions; the solution in the neighborhood of the generation zone was approximately incompressible whereas the solution that was valid farther away comes from the equations of linear acoustics. It was mentioned here that the classical Jantzen–Rayleigh series (see [25]), which is in terms of even powers of M_∞ in steady state hyposonic flow ($S = O(1)$), and in incompressible zone of the sound generation, the odd powers are introduced from M_∞^3 whose presence would not have been suspected but for the powerful matched asymptotic expansions technique. We note that it is possible to find formalized discussion of Viviand–Crow theory in [26], and, in particular, a detailed analysis of the intermediate region between the near field of incompressible flow and the far acoustic field.

However, as $M_\infty \to 0$, the limiting conditions of incompressible flow are not necessarily compatible with the initial conditions (rest) associated with compressible flow condition, and in view of this fact it is necessary to carry out a detailed analysis in the neighborhood of the initial moment t = 0. The study in the neighborhood of the initial moment once again indicates, in the first approximation, the equation of linear acoustics. In the case of the external flow problem, acoustic disturbances associated with the adaptation to the initial conditions, are weakened by the dispersion of acoustic energy in the region whose volume tends to infinity and in view of this fact we cannot expect, at least in the first approximation, the existence of the initial disturbance of the field of almost incompressible flow, valid up to $t = O(1)$. It is impossible to guarantee, a priori, that there will be no higher order effects such that a formal analysis cannot be made. Therefore, it is necessary to make use of the "dispersion" theory for the nonhomogeneous wave equation in the region outside the given enclosed surface (the reader may refer to [27]).

Asymptotic analysis of hyposonic flows poses many problems which, at present, have not been solved. These problems are important for a better understanding of the role of weak compressibility in the mathematical modeling of various physical phenomena in nature. It is clear that we have chosen the terminology hyposonic flow so as to attribute symmetry to the classification of flows based on Mach number (from hyposonic to hypersonic flows). Since the hypersonic aerodynamics is necessary for the study of phenomena at high speeds at very high altitudes (space), dynamics of hyposonic flows appears necessary for a systematic study of flows at very low speeds.

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UNSTEADY FLOW OF A GAS INTO VACUUM THROUGH A PERFORATED PLATE

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The problem of the unsteady flow of a gas into vacuum through a perforated plate is solved within the framework of an approach developed earlier [1]. Two steady orifice flow schemes are used to close the relations at the perforation. The corresponding results of calculations are given for each scheme. The present model, unlike the one proposed in [2, 3], preserves not only the mass flow of gas, but also its total enthalpy.

We direct the x axis along the normal to the perforated plate, which coincides with the plane $x = 0$. It is represented by the hatched strip in Fig. 1a. At time $t \leq 0$ the half-space $x < 0$ is filled with an ideal gas at rest. To the right of the plate is vacuum. At time $t = 0$ the gas begins to flow through the perforation. In terms of its formulation this problem is similar to the problem of the decay of an arbitrary discontinuity at a perforated plate and can be solved within the framework of the approach developed in [1].

If d is a typical linear dimension of the perforation and D is a typical wave propagation velocity, we can assume, as in [1], that for $t \gg d/D$ the flow through the perforation

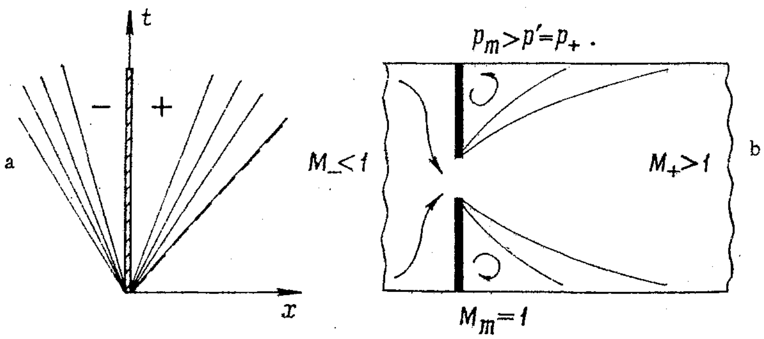

Fig. 1

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